MGRS IN INCOMPLETE INFORMATION SYSTEMS

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Abstract

The original rough set model is concerned primarily with the approximation of sets described by single binary relation on the universe. In the view of granular computing, classical rough set theory is researched by single granulation. The article extends the rough set model based on tolerance relation to incomplete rough set model based on multi-granulations, where the set approximations are defined by using multi tolerance relations on the universe. Its some basic mathematical properties are investigated as well. It is shown that some properties of rough set model based on tolerance relation are special instances of this new model, the approximation measure of a target concept described by using multi-granulations is always better than by using single granulation, which is suitable for describing more accurately the concept and solving problem according to user requirement in incomplete information systems.

Keywords: *Rough set; incomplete information systems; Multi-granulations; Approximation measure.*

1. INTRODUCTION

Rough set theory, proposed by Z. Pawlak [1], has become well established as a mechanism for uncertainty management in a wide variety of applications related to artificial intelligence. Several extensions of rough set model have been proposed in the past, such as variable precision rough set (VPRS) model (see [2]), rough set model based on tolerance relation (see [3]), Bayesian rough set model (see [4]), fuzzy rough set model and rough fuzzy set model (see [5]), etc. In the view of granular computing, however, a general concept described by a set is always characterized via the so-called upper and lower approximations under static granulation, i.e., the concept is depicted by known knowledge induced by single relation on the universe. In practice, we often need to describe the concept through multi-relations on the universe according to user requirement or the target of solving problem. In the view of granular computing (proposed by L. A. Zadeh [6]), an equivalence relation on the universe can be regarded as a granulation, and a partition on the universe can be regarded as a granulation space [7, 8]. For an incomplete information system, similarly, a tolerance relation on the universe can be regard as a granulation, and a cover induced by the relation can be regarded as a granulation space. Several measures in knowledge base closely associated with granular computing, such as knowledge granulation, granulation measure, information entropy and rough entropy, were discussed in [9-11]. An axiomatic approach to measure information granulation in information systems was presented in [12], which provides a basic framework for describing information granulation. On research of rough set method based on multi-granulations, Y. H. Qian and J. Y. Liang brought forward a rough set model based on multi-granulations (**MGRS**) (see [13]), which is established by using multi equivalence relations. The main objective of this paper is an extension of **MGRS**, rough set model based on multi tolerance relations in incomplete information systems.

The paper is organized as follows: the basic concepts of rough set theory and **MGRS** are reviewed in Section 2. In Section 3, rough set method based multi tolerance relations is proposed, its some useful properties are obtained. In Section 4, we conclude the present research.

2. ROUGH SET CONCEPTS

Rough set theory [1] has become well established as a mechanism for uncertainty management in a wide variety of applications related to artificial intelligence.

Let $K=(U,R)$ be an approximation space, where U is a non-empty, finite set called the universe; R is a partition of U, or an equivalence relation on U. $[x]_R$ $(x\in U)$ denotes the equivalence class containing x.

An approximation space $K=(U,R)$ can be regarded as a knowledge base about U. Equivalence class of R is also called elementary set. The equivalence relation R partition the universe U into disjoint subsets. This partition of the universe U induced by R is denoted by U/R.

Given an equivalence relation R on U, and a subset X, we can define a lower approximation of X in U and a upper approximation of X in U by the following

$$\underline{R}X=\bigcup\{x\in U\mid [x]_R\subseteq X\}, \quad (1)$$

and

$$\overline{R}X=\bigcup\{x\in U\mid [x]_R\cap X\neq\varnothing\}. \quad (2)$$

The R-positive region of X is $POS_R(X)=\underline{R}X$, the R-negative region of X is $NEG_R(X)=U-\overline{R}X$, and the boundary or R-borderline region of X is $BN_R(X)=\overline{R}X-\underline{R}X$. X is called R-definable if and only if $\overline{R}X=\underline{R}X$. Otherwise $\overline{R}X\neq\underline{R}X$ and X is rough with respect to R.

Let $K=(U,R)$ be an approximation space, $X\in U$ a subset on U. The approximation measure $\alpha_R(X)$ is defined as

$$\alpha_R(X)=\frac{|\underline{R}X|}{|\overline{R}X|}, \quad (3)$$

where $X\neq\varnothing$, $|X|$ denotes the cardinality of set X.

Definition 1(see [13]). Let $K=(U,\mathbf{R})$ be a knowledge base, $X\subseteq U$, $P_1,P_2,\cdots,P_m\in\mathbf{R}$, we can define a lower approximation of X and a upper approximation of X related to $P_1,P_2,\cdots,P_m$ by the following

$$\underline{\sum_{i=1}^{m}P_i}X=\bigcup\{x\mid [x]_{P_i}\subseteq X, i\leq m\}, \quad (4)$$

and

$$\overline{\sum_{i=1}^{m}P_i}X=\sim\underline{\sum_{i=1}^{m}P_i}(\sim X). \quad (5)$$

The boundary is defined as

$$BN_{\sum_{i=1}^{m}P_i}(X)=\overline{\sum_{i=1}^{m}P_i}X-\underline{\sum_{i=1}^{m}P_i}X.$$

3. MGRS IN INCOMPLETE INFORMATION SYSTEMS

An information system is a pair $S=(U,A)$, where,

(1) U is a non-empty finite set of objects;

(2) A is a non-empty finite set of attributes;

(3) for every $a\in A$, there is a mapping a, $a:U\to V_a$, where V_a is called the value set of a.

If V_a contains a null value for at least one attribute $a\in A$, then S is called an incomplete information system, otherwise it is complete (see [3]). Further on, we will denote the null value by *.

Let $S=(U,A)$ be an incomplete information system, $P\subseteq A$ an attribute set. We define a binary relation on U as follows

$$SIM(P)=\{(u,v)\in U\times U\mid \forall a\in P, a(u)=a(v) \text{ or } a(u)=* \text{ or } a(v)=*\}.$$

In fact, $SIM(P)$ is a tolerance relation on U, the concept of a tolerance relation has a wide variety of applications in classification (see [3]).

It can be shown that $SIM(P)=\bigcap_{a\in P}SIM(\{a\})$.

Let $S_P(u)$ denote the set $\{v\in U\mid (u,v)\in SIM(P)\}$. $S_P(u)$ is the maximal set of objects which are possibly indistinguishable by P with u.

Let $U/SIM(P)$ denote the family sets $\{S_P(u)\mid u\in U\}$, the classification or the knowledge induced by P. A member $S_P(u)$ from $U/SIM(P)$ will be called a tolerance class or an information granule. It should be noticed that the tolerance classes in $U/SIM(P)$ do not constitute a partition of U in general. They constitute a cover of U, i.e., $S_P(u)\neq\varnothing$ for every $u\in U$, and $\bigcup_{u\in U}S_P(u)=U$.

Now we define a partial order on the set of all classifications of U. Let $S=(U,A)$ be an incomplete information system, $P,Q\subseteq A$. We say that Q is coarser than P (or P is finer than Q), denoted by $P\preceq Q$, if and only if $S_P(u_i)\subseteq S_Q(u_i)$, and $\forall i\in\{1,2,\cdots,|U|\}$. If $P\preceq Q$ and $P\neq Q$, we say that Q is strictly coarser than P (P is strictly finer than Q) and denoted by $P\prec Q$ (see [10]).

In fact, $P \prec Q \Leftrightarrow$ for $\forall i \in \{1, 2, \cdots, |U|\}$, we have that $S_P(u_i) \subseteq S_Q(u_i)$, and $\exists j \in \{1, 2, \cdots, |U|\}$, such that $S_P(u_j) \subset S_Q(u_j)$.

In the view of granular computing, the approximation of a set in incomplete information systems is described by using a single tolerance relation (granulation) on the universe. Simply, we discuss firstly the approximation of a set by using two tolerance relations on the universe, i.e., the target concept is described by two granulation spaces.

Definition 2. Let $S=(U,A)$ be an incomplete information system, $P,Q \subseteq A$ two attribute subsets, $X \subseteq U$, we define a lower approximation of X and a upper approximation of X in U by the following

$$\underline{P+Q}X = \bigcup\{x \mid SIM_P(x) \subseteq X \; or \; SIM_Q(x) \subseteq X\}, \quad (6)$$

and

$$\overline{P+QX} = \sim \underline{P+Q}(\sim X). \quad (7)$$

We will illuminate the rough set approximation based on multi-granulations and the difference between the rough sets method and the rough set model based on single tolerance relation by the following example.

Example 1. Let $S=(U,A)$ be an incomplete information system, $P,Q \subseteq A$ two attribute subsets, $X \subseteq U$, where

$U = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$,

$X = \{e_1, e_2, e_6, e_8\}$.

$U/SIM(P) = \{\{e_1,e_7\},\{e_2,e_3,e_4,e_5,e_6\},\{e_2,e_3,e_4,e_5,e_6\},$
$\{e_2,e_3,e_4,e_5,e_6\},\{e_2,e_3,e_4,e_5,e_6\},\{e_2,e_3,e_4,e_5,e_6\},$
$\{e_1,e_7\},\{e_8\}\}$,

$U/SIM(Q) = \{\{e_1,e_2\},\{e_1,e_2\},\{e_3,e_4,e_5\},\{e_3,e_4,e_5\},$
$\{e_3,e_4,e_5\},\{e_6,e_7,e_8\},\{e_6,e_7,e_8\},\{e_6,e_7,e_8\}\}$,

$U/SIM(P \cup Q) = \{\{e_1\},\{e_2\},\{e_3,e_4,e_5\},\{e_3,e_4,e_5\},\{e_3,e_4,e_5\},$
$\{e_6\},\{e_7\},\{e_8\}\}$.

By computing, we have that

$$\underline{P+Q}X = \bigcup\{x \mid SIM_P(x) \subseteq X \; or \; SIM_Q(x) \subseteq X\},$$
$$= \{e_8\} \cup \{e_1,e_2\}$$
$$= \{e_1,e_2,e_8\},$$

$$\overline{P+QX} = \sim \underline{P+Q}(\sim X).$$
$$= \sim \{\emptyset \cup \{e_3,e_4,e_5\}$$
$$= \{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\} \cap \{e_1,e_2,e_6,e_7,e_8\}$$
$$= \{e_1,e_2,e_6,e_7,e_8\}.$$

$$\underline{P \cup Q}X = \{e_1,e_2,e_6,e_8\},$$
$$\overline{P \cup QX} = \{e_1,e_2,e_6,e_8\}.$$

Obviously, it follows from above computation that

$$\underline{P+Q}X \neq \underline{P \cup Q}X,$$
$$\overline{P+QX} \neq \overline{P \cup QX}.$$

Proposition 1. Let $S=(U,A)$ be an incomplete inform -ation system, $P,Q \subseteq A$ two attribute subsets, $X \subseteq U$. Then

1) $\underline{P+Q}X \subseteq \underline{P \cup Q}X$,

2) $\overline{P+QX} \supseteq \overline{P \cup QX}$.

Proof.

1) For any $x \in \underline{P+Q}X$, from Definition 2, it follows that $SIM_P(x) \subseteq X$ or $SIM_Q(x) \subseteq X$. Hence, we have that $x \in SIM_P(x) \cap SIM_Q(x)$. But $SIM_P(x) \cap SIM_Q(x) \subseteq SIM_{P \cup Q}(x)$ for $x \in U$, therefore $x \in SIM_{P \cup Q}(x)$, i.e., $\underline{P+Q}X \subseteq \underline{P \cup Q}X$.

2) From rough set model based on tolerance relation, we know $\overline{P \cup QX} = \sim \underline{P \cup Q}(\sim X)$. Since 1), we get that

$$\overline{P \cup QX} = \sim \underline{P \cup Q}(\sim X)$$
$$\subseteq \sim \underline{P+Q}(\sim X)$$
$$= \overline{P+QX},$$

that is $\overline{P+QX} \supseteq \overline{P \cup QX}$.

This completes the proof. □

Directly from the definition of approximations we can get the following properties of the lower and the upper approximations.

Proposition 2. Let $S=(U,A)$ be an incomplete inform -ation system, $P,Q \subseteq A$ two attribute subsets, $X \subseteq U$. Then the following properties hold

1) $\underline{P+Q}X \subseteq X \subseteq \overline{P+QX}$;

2) $\underline{P+Q}\emptyset = \overline{P+Q\emptyset} = \emptyset$;

$\underline{P+Q}U = \overline{P+QU} = U$;

3) $\underline{P+Q}(\sim X) = \sim \overline{P+QX}$;

$\overline{P+Q}(\sim X) = \sim \underline{P+Q}X$;

4) $\underline{P+Q}X = \underline{P}X \cup \underline{Q}X$;

5) $\overline{P+Q}X = \overline{P}X \cap \overline{Q}X$;

6) $\underline{P+Q}X = \underline{Q+P}X$, $\overline{P+Q}X = \overline{Q+P}X$.

Proof. Their proofs are omitted here.

In order to discover the relationship the approximation of a single set and the approximation of two sets described by using two granulations, the following properties are given.

Proposition 3. Let $S=(U,A)$ be an incomplete inform -ation system, $P,Q\subseteq A$two attribute subsets, $X,Y\subseteq U$. Then the following properties hold,

1) $\underline{P+Q}(X\cap Y) = (\underline{P}X\cap\underline{P}Y)\cup(\underline{Q}X\cap\underline{Q}Y)$;

2) $\overline{P+Q}(X\cup Y) = (\overline{P}X\cup\overline{P}Y)\cap(\overline{Q}X\cup\overline{Q}Y)$;

3) $\underline{P+Q}(X\cap Y)\subseteq \underline{P+Q}(X)\cap\underline{P+Q}(Y)$;

4) $\overline{P+Q}(X\cup Y)\supseteq \overline{P+Q}(X)\cup\overline{P+Q}(Y)$;

5) $X\subseteq Y \Rightarrow \underline{P+Q}X\subseteq\underline{P+Q}Y$;

6) $X\subseteq Y \Rightarrow \overline{P+Q}X\subseteq\overline{P+Q}Y$;

7) $\underline{P+Q}(X\cup Y)\supseteq \underline{P+Q}X\cup\underline{P+Q}Y$;

8) $\overline{P+Q}(X\cap Y)\subseteq\overline{P+Q}X\cap\overline{P+Q}Y$.

Proof. If $P=Q$, then they are the same as upper approximation and lower approximation of rough set model based on tolerance relation (see [3]), and hence 1)-8) hold.

If $P\neq Q$, we give their proving as follows.

1) $\underline{P+Q}(X\cap Y) = \underline{P}(X\cap Y)\cup\underline{Q}(X\cap Y)$

$$= (\underline{P}X\cap\underline{P}Y)\cup(\underline{Q}X\cap\underline{Q}Y).$$

2) $\overline{P+Q}(X\cup Y)=\overline{P}(X\cup Y)\cap\overline{Q}(X\cup Y)$

$$= (\overline{P}X\cup\overline{P}Y)\cap(\overline{Q}X\cup\overline{Q}Y).$$

3) It follows from 1) that

$$\underline{P+Q}(X\cap Y) = (\underline{P}X\cap\underline{P}Y)\cup(\underline{Q}X\cap\underline{Q}Y)$$
$$= ((\underline{P}X\cap\underline{P}Y)\cup\underline{Q}X)\cap((\underline{P}X\cap\underline{P}Y)\cup\underline{Q}Y)$$
$$=((\underline{P}X\cup\underline{Q}X)\cap(\underline{P}Y\cup\underline{Q}X))\cap((\underline{P}X\cup\underline{Q}Y)\cap(\underline{P}Y\cup\underline{Q}Y))$$
$$=\underline{P+Q}X\cap\underline{P+Q}Y\cap((\underline{P}X\cup\underline{Q}Y)\cap(\underline{P}Y\cup\underline{Q}X))$$
$$\subseteq \underline{P+Q}X\cap\underline{P+Q}Y.$$

4) It follows from 2) that

$$\overline{P+Q}(X\cup Y) = (\overline{P}X\cup\overline{P}Y)\cap(\overline{Q}X\cup\overline{Q}Y)$$
$$=((\overline{P}X\cup\overline{P}Y)\cap\overline{Q}X)\cup((\overline{P}X\cup\overline{P}Y)\cap\overline{Q}Y)$$
$$=((\overline{P}X\cap\overline{Q}X)\cup(\overline{P}Y\cap\overline{Q}X))\cup((\overline{P}X\cap\overline{Q}Y)\cup(\overline{P}Y\cap\overline{Q}Y))$$
$$=\overline{P+Q}X\cup\overline{P+Q}Y\cup((\overline{P}X\cap\overline{Q}Y)\cup(\overline{P}Y\cap\overline{Q}X))$$
$$\supseteq\overline{P+Q}X\cup\overline{P+Q}Y.$$

5) If $X\subseteq Y$, then $X\cap Y = X$. It follows from 3) that

$$\underline{P+Q}(X\cap Y)=\underline{P+Q}X\subseteq\underline{P+Q}X\cap\underline{P+Q}Y$$
$$\Rightarrow \underline{P+Q}X = \underline{P+Q}X\cap\underline{P+Q}Y$$
$$\Rightarrow \underline{P+Q}X\subseteq\underline{P+Q}Y.$$

6) If $X\subseteq Y$, then $X\cup Y = Y$. It follows from 4) that

$$\overline{P+Q}(X\cup Y)=\overline{P+Q}Y\supseteq\overline{P+Q}X\cup\overline{P+Q}Y$$
$$\Rightarrow \overline{P+Q}Y=\overline{P+Q}X\cup\overline{P+Q}Y$$
$$\Rightarrow \overline{P+Q}X\subseteq\overline{P+Q}Y.$$

7) It is clear that $X\subseteq X\cup Y$, $Y\subseteq X\cup Y$. It follows that

$$\underline{P+Q}X\subseteq\underline{P+Q}(X\cup Y),$$
$$\underline{P+Q}Y\subseteq\underline{P+Q}(X\cup Y).$$

Hence $\underline{P+Q}X\cup\underline{P+Q}Y\subseteq\underline{P+Q}(X\cup Y)$;

8) It is clear that $X\cap Y\subseteq X$, $X\cap Y\subseteq Y$. It follows that

$$\overline{P+Q}(X\cap Y)\subseteq\overline{P+Q}X,$$
$$\overline{P+Q}(X\cap Y\subseteq\overline{P+Q}Y).$$

Hence $\overline{P+Q}(X\cap Y)\subseteq\overline{P+Q}X\cap\overline{P+Q}Y$.

This completes the proof. □

Based on above conclusions, we here extend to rough set model based on single tolerance relation to rough set model based on multi-granulations in an incomplete information system, where the set approximations are defined by using multi tolerance relations on the universe.

Definition 3. Let $S=(U,A)$ be an incomplete inform - ation system, $X\subseteq U$, $P_1,P_2,\cdots,P_m\subseteq A$, we can define a lower approximation of X and a upper approximation of X related to $P_1,P_2,\cdots,P_m$ by the following

$$\underline{\sum\nolimits_{i=1}^{m} P_i}X = \bigcup\{x \mid SIM_{P_i}(x)\subseteq X, i\le m\}, \quad (8)$$

and

$$\overline{\sum_{i=1}^{m} P_i} X = \sim \underline{\sum_{i=1}^{m} P_i}(\sim X). \qquad (9)$$

Directly from the definition of approximations we can get the following properties of the lower and the upper approximations.

Proposition 4. Let $S=(U,A)$ be an incomplete informa -tion system, $X \subseteq U$. Then the following properties hold

1) $\underline{\sum_{i=1}^{m} P_i} X = \bigcup_{i=1}^{m} \underline{P_i} X$;

2) $\overline{\sum_{i=1}^{m} P_i} X = \bigcap_{i=1}^{m} \overline{P_i} X$;

3) $\overline{\sum_{i=1}^{m} P_i}(\sim X) = \sim \underline{\sum_{i=1}^{m} P_i} X$;

4) $\underline{\sum_{i=1}^{m} P_i}(\sim X) = \sim \overline{\sum_{i=1}^{m} P_i} X$.

Proof. 1) From the formula (4) in Proposition 2, it can be proved.

2) From (9) and 1), we have that

$$\overline{\sum_{i=1}^{m} P_i} X = \sim \underline{\sum_{i=1}^{m} P_i}(\sim X) = \sim \bigcup_{i=1}^{m} \underline{P_i}(\sim X)$$
$$= \sim \bigcup_{i=1}^{m} (\sim \overline{P_i} X) = \bigcap_{i=1}^{m} \overline{P_i} X .$$

3) It is straightforward from (9).

4) Let $X = \sim X$ in (9), we have that

$$\underline{\sum_{i=1}^{m} P_i}(\sim X) = \sim \overline{\sum_{i=1}^{m} P_i} X .$$

This completes the proof. □

Proposition 5. Let $S=(U,A)$ be an incomplete inform -ation system, $X_1, X_2, \cdots, X_n \subseteq U$ be n subsets on U, $P_1, P_2, \cdots, P_m \subseteq A$, the following properties hold

1) $\underline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) = \bigcup_{i=1}^{m} (\bigcap_{j=1}^{n} \underline{P_i} X_j)$;

2) $\overline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) = \bigcap_{i=1}^{m} (\bigcup_{j=1}^{n} \overline{P_i} X_j)$;

3) $\underline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) \subseteq \bigcap_{j=1}^{n} (\underline{\sum_{i=1}^{m} P_i} X_j)$;

4) $\overline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) \supseteq \bigcup_{j=1}^{n} (\overline{\sum_{i=1}^{m} P_i} X_j)$;

5) $\underline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) \supseteq \bigcup_{j=1}^{n} (\underline{\sum_{i=1}^{m} P_i} X_j)$;

6) $\overline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) \subseteq \bigcap_{j=1}^{n} (\overline{\sum_{i=1}^{m} P_i} X_j)$.

Proof. Similar to Proposition 3, we can prove the following properties.

1) $\underline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) = \bigcup_{i=1}^{m} \underline{P_i}(\bigcap_{j=1}^{n} X_j)$
$= \bigcup_{i=1}^{m} (\bigcap_{j=1}^{n} \underline{P_i} X_j)$.

2) $\overline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) = \bigcap_{i=1}^{m} \overline{P_i}(\bigcup_{j=1}^{n} X_j)$
$= \bigcap_{i=1}^{m} (\bigcup_{j-1}^{n} \overline{P_i} X_j)$.

3) $\underline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) = \bigcup_{i=1}^{m} (\bigcap_{j=1}^{n} \underline{P_i} X_j)$
$= \bigcap_{j=1}^{n} (\bigcup_{i=1}^{m} \underline{P_i} X_j) \cap \cdots$
$= \bigcap_{j=1}^{n} (\underline{\sum_{i=1}^{m} P_i} X_j) \cap \cdots$
$\subseteq \bigcap_{j=1}^{n} (\underline{\sum_{i=1}^{m} P_i} X_j)$.

4) $\overline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) = \bigcap_{i=1}^{m} (\bigcup_{j=1}^{n} \overline{P_i} X_j)$
$= \bigcup_{j=1}^{n} (\bigcap_{i=1}^{m} \overline{P_i} X_j) \cup \cdots$
$= \bigcup_{j=1}^{n} (\overline{\sum_{i=1}^{m} P_i} X_j) \cup \cdots$
$\supseteq \bigcup_{j=1}^{n} (\overline{\sum_{i=1}^{m} P_i} X_j)$.

5) It follows from $\forall X_j \subseteq \bigcup_{j=1}^{n} X_j$ that

$$\underline{\sum_{i=1}^{m} P_i} X_j \subseteq \underline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j).$$

Hence, we have that

$$\underline{\sum_{i=1}^{m} P_i}(\bigcup_{j=1}^{n} X_j) \supseteq \bigcup_{j=1}^{n} (\underline{\sum_{i=1}^{m} P_i} X_j).$$

6) It follows from $\bigcap_{j=1}^{n} X_j \subseteq X_j \, (j \in \{1,2,\cdots,n\})$ that $\overline{\sum_{i=1}^{m} P_i} X_j \supseteq \overline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j)$. Hence, we have $\overline{\sum_{i=1}^{m} P_i}(\bigcap_{j=1}^{n} X_j) \subseteq \bigcap_{j=1}^{n} (\overline{\sum_{i=1}^{m} P_i} X_j)$.

This completes the proof. □

4. CONCLUSIONS

The main objective of this paper is an extension of rough set model based on tolerance relation under static granulation, rough set model based on multi-granulations (**MGRS**) in incomplete information systems, where the approximations of sets are defined by using multi tolerance relations on the universe. These tolerance relations are chosen according to user requirement or the target of solving problem. The method has some useful properties. In particular, some properties of original rough set model based on tolerance relation are special instances of **MGRS** in incomplete information systems, and approximation measure of a target concept described by using multi-granulations is always better than by using single granulation.

Presented approach appears to be well suited for data mining applications where the acquisition of decision rules with high approximation measure, and further studying rough set theory. Further research is planned to evaluate the MGRS method in comparison to original rough set approaches, and to extend other rough set methods.

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