Consistency and Fuzziness in Ordered Decision Tables

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Abstract. In this paper, we focus on how to measure the consistency of an ordered decision table and the fuzziness of an ordered rough set and an ordered rough classification in the context of ordered information systems. The membership function of an object is defined through using the dominance class including itself. Based on the membership function, we introduce a consistency measure to assess the consistency of an ordered decision table and define two fuzziness measures to compute the fuzziness of an ordered rough set and an ordered rough classification. Several examples are employed to illustrate their mechanisms as well. These results will be helpful for understanding the uncertainty in ordered information systems and ordered decision tables.

Keywords: Ordered decision table, Consistency, Fuzziness.

1 Introduction

Rough set theory, introduced by Pawlak [1, 2], has been conceived as a tool to conceptualize and analyze various types of data. It can be used in the attribute-value representation model to describe the dependencies among attributes and evaluate the significance of attributes and derive decision rules. It has important applications to intelligence decision and cognitive sciences, as a tool for dealing with vagueness and uncertainty of facts, and in classification [3-8]. Rough-set-based data analysis starts from a data table, called information systems. The information systems contains data about objects of interest, characterized by a finite set of attributes [9-14].

The original rough sets theory does not consider attributes with preference-ordered domains, that is, criteria. However, in many real situations, we are often faced with the problems in which the ordering of properties of the considered attributes plays a crucial role. One such type of problem is the ordering of objects. For this reason, Greco, Matarazzo, and Slowinski [15, 16] proposed an extension of rough set theory, called the dominance-based rough sets approach (DRSA) to take into account the ordering properties of criteria. This innovation is mainly based on substitution of the indiscernibility relation by a dominance relation.

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Because the notion of consistency degree [1] is defined for a decision table, in some sense, it could be regarded as measures for evaluating the decision performance of a decision table [17, 18]. Nevertheless, the consistency degree has some limitations. For instance, the consistency of a decision table could not be well depicted by the consistency degree when its value achieve zero. As we know, the fact that consistency degree is equal to zero only implies that there is no decision rule with the certainty of one in the decision table. Hence, the consistency degree of a decision table cannot give elaborate depictions of the consistency for a given decision table. Therefore, we introduced three new measures to assess the entire decision performance of a decision-rule set extracted from a complete/incomplete decision table [18, 19]. So far, however, how to assess the consistency of an ordered decision table has not been reported. In addition, like classical rough set theory, there exist some fuzziness of an ordered rough set and an ordered rough classification in the dominance-based rough sets approach.

The rest of the paper is organized as follows. Some basic concepts of ordered information systems and ordered decision tables are briefly reviewed in Section 2. In Section 3, how to measure the consistencies of a set and an ordered decision table are investigated. In Section 4, we propose fuzziness measures of an ordered rough set and an ordered rough classification in an ordered decision table. Section 5 concludes this paper with some remarks.

2 Preliminaries

In this section, we recall some basic concepts of ordered information systems and ordered decision tables.

An information system (IS) is an quadruple $S = (U, AT, V, f)$, where U is a finite nonempty set of objects and AT is a finite nonempty set of attributes, $V = \bigcup_{a \in AT} V_a$ and V_a is a domain of attribute a, $f : U \times AT \rightarrow V$ is a total function such that $f(x, a) \in V_a$ for every $a \in AT$, $x \in U$, called an information function. A decision table is a special case of an information system in which, among the attributes, we distinguish one called a decision attribute. The other attributes are called condition attributes. Therefore, $S = (U, C \cup d, V, f)$ and $C \cap d = \emptyset$, where the set C is called the condition attributes and d is called the decision attribute.

If the domain (scale) of a condition attribute is ordered according to a decreasing or increasing preference, then the attribute is a criterion.

Definition 1.[20] A decision table is called an ordered decision table (ODT) if all condition attributes are criterions.

It is assumed that the domain of a criterion $a \in AT$ is completely pre-ordered by an outranking relation $\succeq_a$; $x \succeq_a y$ means that x is at least as good as (outranks) y with respect to criterion a. In the following, without any loss of generality, we consider a condition criterion having a numerical domain, that is, $V_a \subseteq \mathbf{R}$ ($\mathbf{R}$ denotes the set of real numbers) and being of type gain, that is,

$x \succeq_a y \Leftrightarrow f(x,a) \geq f(y,a)$ (according to increasing preference) or $x \succeq_a y \Leftrightarrow f(x,a) \leq f(y,a)$ (according to decreasing preference), where $a \in AT$, $x, y \in U$. For a subset of attributes $B \subseteq C$, we define $x \succeq_B y \Leftrightarrow \forall a \in B, f(x,a) \geq f(y,a)$. In other words, x is at least as good as y with respect to all attributes in B. In general, the domain of the condition criterion may be also discrete, but the preference order between its values has to be provided.

In a given ordered information system, we say that x dominates y with respect to $B \subseteq C$ if $x \succeq_B y$, and denoted by $xR_B^{\geqq}y$. That is $R_B^{\geqq} = \{(y,x) \in U \times U \mid y \succeq_B x\}$. Obviously, if $(y,x) \in R_B^{\geqq}$, then y dominates x with respect to B.

Let B_1 be attributes set according to increasing preference and B_2 attributes set according to decreasing preference, hence $B = B_1 \cup B_2$. The granules of knowledge induced by the dominance relation $R_B^{\geqq}$ are the set of objects dominating x, that is

$$[x]_B^{\geqq} = \{y \mid f(y,a_1) \geq f(x,a_1)(\forall a_1 \in B_1) \text{ and } f(y,a_2) \leq f(x,a_2)(\forall a_2 \in B_2)\}$$
$$= \{y \in U \mid (y,x) \in R_B^{\geqq}\}$$

and the set of objects dominated by x,

$$[x]_B^{\leqq} = \{y \mid f(y,a_1) \leq f(x,a_1)(\forall a_1 \in B_1) \text{ and } f(y,a_2) \geq f(x,a_2)(\forall a_2 \in B_2)\}$$
$$= \{y \in U \mid (x,y) \in R_B^{\geqq}\},$$

which are called the B-dominating set and the B-dominated set with respect to $x \in U$, respectively.

Let $U/R_B^{\geqq}$ denote classification on the universe, which is the family set $\{[x]_B^{\geqq} \mid x \in U\}$. Any element from $U/R_B^{\geqq}$ will be called a dominance class with respect to B. Dominance classes in $U/R_B^{\geqq}$ do not constitute a partition of U in general. They constitute a covering of U.

3 Consistency of an Ordered Decision Table

In this section, we deal with how to measure the consistency of an ordered decision table.

Let $S = (U, AT)$ be an ordered information system, $P, Q \subseteq A$, $U/R_P^{\geqq} = \{[x_1]_P^{\geqq}, [x_2]_P^{\geqq}, \cdots, [x_{|U|}]_P^{\geqq}\}$ and $U/R_Q^{\geqq} = \{[x_1]_Q^{\geqq}, [x_2]_Q^{\geqq}, \cdots, [x_{|U|}]_Q^{\geqq}\}$. We define a partial relation $\preceq$ as follows: $P \preceq Q \Leftrightarrow [x_i]_P^{\geqq} \subseteq [x_i]_Q^{\geqq}$ for any $x_i \in U$, where $[x_i]_P^{\geqq} \in U/R_P^{\geqq}$ and $[x_i]_Q^{\geqq} \in U/R_Q^{\geqq}$. If $P \preceq Q$, we say that Q is coarser than P (or P is finer than Q).

Let $S = (U, C \cup d)$ be an ordered decision table, $U/R_C^{\geqq} = \{[x_1]_C^{\geqq}, [x_2]_C^{\geqq}, \cdots, [x_{|U|}]_C^{\geqq}\}$ and $U/R_d^{\geqq} = \{[x_1]_d^{\geqq}, [x_2]_d^{\geqq}, \cdots, [x_{|U|}]_d^{\geqq}\}$. If $C \preceq d$, then S is said to be a consistent ordered decision table; otherwise, S is said to be inconsistent.

Firstly, we investigate the consistency of the dominance class $[x_i]_C^{\geqq}$ ($i \in \{1, 2, \cdots, |U|\}$) with respect to d in an ordered decision table.

Let $S = (U, C \cup d)$ be an ordered decision table, $[x_i]_C^{\geqq} \in U/R_C^{\geqq}$ a dominance class and $U/R_d^{\geqq} = \{[x_i]_d^{\geqq} : x_i \in U\}$. For any object $x \in U$, the membership function of x in the dominance class $[x_i]_C^{\geqq}$ is defined as

$$\delta_{[x_i]_C^{\geq}}(x) = \begin{cases} \frac{|[x_i]_C^{\geq} \cap [x]_d^{\geq}|}{|[x_i]_C^{\geq}|}, & if\ x = x_i; \\ 0, & if\ x \neq x_i. \end{cases} \quad (1)$$

Where $\delta_{[x_i]_C^{\geq}}(x)$ denotes a fuzzy concept.

If $\delta_{[x_i]_C^{\geq}}(x) = 1$, then the dominance class $[x_i]_C^{\geq}$ can be said to be consistent with respect to d. In other words, if $[x_i]_C^{\geq}$ is a consistent set with respect to d, then $[x_i]_C^{\geq} \subseteq [x_i]_d^{\geq}$. This generates a fuzzy set $F_{[x_i]_C^{\geq}}^d = \{(x, \delta_{[x_i]_C^{\geq}}(x)) \mid x \in U\}$ on the universe U.

Definition 2. Let $S = (U, C \cup d)$ be an ordered decision table, $[x_i]_C^{\geq} \in U/R_C^{\geq}$ a dominance class and $U/R_d^{\geq} = \{[x_1]_d^{\geq}, [x_2]_d^{\geq}, \cdots, [x_{|U|}]_d^{\geq}\}$. A consistency measure of $[x_i]_C^{\geq}$ with respect to d is defined as

$$C([x_i]_C^{\geq}, d) = \sum_{x \in U} \delta_{[x_i]_C^{\geq}}(x), \quad (2)$$

where $0 \leq C([x_i]_C^{\geq}, d) \leq 1$.

Proposition 1. The consistency measure of a consistent dominance class in an ordered decision table is one.

In the following, based on the above discussion, we research the consistency between the condition part and the decision part in an ordered decision table.

Definition 3. Let $S = (U, C \cup d)$ be an ordered decision table, $U/R_C^{\geq} = \{[x_1]_C^{\geq}, [x_2]_C^{\geq}, \cdots, [x_{|U|}]_C^{\geq}\}$ and $U/R_d^{\geq} = \{[x_1]_d^{\geq}, [x_2]_d^{\geq}, \cdots, [x_{|U|}]_d^{\geq}\}$. A consistency measure of C with respect to d is defined as

$$C(C, d) = \frac{1}{|U|} \sum_{i=1}^{|U|} \sum_{x \in U} \delta_{[x_i]_C^{\geq}}(x), \quad (3)$$

where $0 \leq C(C, d) \leq 1$ and $\delta_{[x_i]_C^{\geq}}(x)$ is the membership function of $x \in U$ in the dominance class $[x_i]_C^{\geq}$.

Example 1. An ODT is presented in Table 1, where $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $C = \{a_1, a_2, a_3\}$.

Table 1. An ordered decision table

U	a_1	a_2	a_3	d
x_1	1	2	1	1
x_2	3	2	2	2
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	1
x_6	3	2	3	2

In this table, from the definition of dominance classes, one can obtain that the dominance classes determined by C are

$[x_1]_C^{\geq} = \{x_1, x_2, x_5, x_6\}$, $[x_2]_C^{\geq} = \{x_2, x_5, x_6\}$, $[x_3]_C^{\geq} = \{x_2, x_3, x_4, x_5, x_6\}$, $[x_4]_C^{\geq} = \{x_4, x_6\}$, $[x_5]_C^{\geq} = \{x_5\}$, $[x_6]_C^{\geq} = \{x_6\}$;

and the dominance classes determined by d are

$[x_1]_d^{\geq} = [x_3]_d^{\geq} = [x_5]_d^{\geq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $[x_2]_d^{\geq} = [x_4]_d^{\geq} = [x_6]_d^{\geq} = \{x_2, x_4, x_6\}$.

From formula (1), one has that

$C([x_1]_C^{\geq}, d) = 1$, $C([x_2]_C^{\geq}, d) = \frac{2}{3}$, $C([x_3]_C^{\geq}, d) = 1$, $C([x_4]_C^{\geq}, d) = 1$, $C([x_5]_C^{\geq}, d) = 1$ and $C([x_6]_C^{\geq}, d) = 1$. Therefore,

$C(C, d) = \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{x\in U}\delta_{[x_i]_C^{\geq}}(x) = \frac{1}{6}(1 + \frac{2}{3} + 1 + 1 + 1 + 1) = \frac{17}{18}$.

Proposition 2. The consistency measure of a consistent ordered decision table is one.

Proof. Let $S = (U, C \cup d)$ be an ordered decision table, $U/R_C^{\geq} = \{[x_1]_C^{\geq}, [x_2]_C^{\geq}, \cdots, [x_{|U|}]_C^{\geq}\}$ and $U/R_d^{\geq} = \{[x_1]_d^{\geq}, [x_2]_d^{\geq}, \cdots, [x_{|U|}]_d^{\geq}\}$. If S is consistent, then, for any $x_i \in U$, one has $[x_i]_C^{\geq} \subseteq [x_i]_d^{\geq}$. Hence, when $x = x_i$, we have $\delta_{[x_i]_C^{\geq}}(x) = \frac{|[x_i]_C^{\geq} \cap [x_i]_d^{\geq}|}{|[x_i]_C^{\geq}|} = \frac{|[x_i]_C^{\geq}|}{|[x_i]_C^{\geq}|} = 1$; otherwise, $\delta_{[x_i]_C^{\geq}}(x) = 0$. Therefore,

$$C(C, d) = \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{x\in U}\delta_{[x_i]_C^{\geq}}(x) = \frac{1}{|U|}\sum_{i=1}^{|U|}(1 \cdot 1 + (|U| - 1) \cdot 0) = 1.$$

Thus, the consistency measure of a consistent ordered decision table is one.

4 Fuzziness of an Ordered Rough Set and an Ordered Rough Classification

In this section, we present fuzziness measures of an ordered rough set and an ordered rough classification in an ordered decision table.

In the literature, Greco et al. [15, 16] proposed the rough set theory for multi-criteria decision analysis. For any $X \subseteq U$ and $B \subseteq C$, the lower and upper approximation of X with respect to the dominance relation $R_B^{\geq}$ are defined as $\underline{R_B^{\geq}}(X) = \{x \in U \mid [x]_B^{\geq} \subseteq X\}$ and $\overline{R_B^{\geq}}(X) = \{[x]_B^{\geq} \mid [x]_B^{\geq} \cap X \neq \emptyset\}$. Unlike classical rough set theory, one can easily notice the properties $\underline{R_B^{\geq}}(X) = \{[x]_B^{\geq} \mid [x]_B^{\geq} \subseteq X\}$ and $\overline{R_B^{\geq}}(X) = \{[x]_B^{\geq} \mid [x]_B^{\geq} \cap X \neq \emptyset\}$ do not hold.

Let $S = (U, AT)$ be an ordered information system and $X \subseteq U$. For any object $x \in U$, the membership function of x in X is defined as

$$\mu_X(x) = \frac{|[x]_{AT}^{\geq} \cap X|}{|[x]_{AT}^{\geq}|} \tag{4}$$

where $\mu_X(u)$ $(0 \leq \mu_X(u) \leq 1)$ represents a fuzzy concept. It can generate a fuzzy set $F_X^{AT} = \{(x, \mu_X(x)) \mid x \in U\}$ on the universe U. Based on this membership function, one can define a fuzzy measure of a given rough set induced by the attribute set AT as follows.

Definition 4. Let $S = (U, A)$ be an ordered information system and $X \subseteq U$. A fuzziness measure of the rough set X is defined as

$$E(F_X^{AT}) = \sum_{i=1}^{|U|} \mu_X(x_i)(1 - \mu_X(x_i)). \tag{5}$$

Proposition 3. The fuzziness measure of a crisp set equals zero in an ordered information system.

Proof. Let X be a crisp set in the ordered information system $S = (U, AT)$, then $\underline{R_{AT}^{\geq}}(X) = \overline{R_{AT}^{\geq}}(X)$. Hence, for any $x \in U$, one can get that if $x \in \underline{R_{AT}^{\geq}}(X)$, then $[x]_{AT}^{\geq} \subseteq X$, thus $\mu_X(x) = 1$; and if $x \notin \underline{R_{AT}^{\geq}}(X)$, then $x \notin \overline{R_{AT}^{\geq}}(X)$, i.e., $[x]_{AT}^{\geq} \cap X = \emptyset$, thus $\mu_X(x) = 0$. Therefore, one has that $\mu_X(x)(1 - \mu_X(x)) = 0$, that is $E(F_X^{AT}) = 0$. This completes the proof.

Proposition 4. The fuzziness measure of a rough set is the same as that of its complement set in an ordered information system.

Proof. Let X be a rough set in the ordered information system $S = (U, AT)$ and X^c is its complement set on the universe U, i.e., $X^c = U - X$. For any $x \in U$, one has that

$$\mu_X(x) + \mu_{X^c}(x) = \frac{|X \cap [x]_{AT}^{\geq}|}{|[x]_{AT}^{\geq}|} + \frac{|X^c \cap [x]_{AT}^{\geq}|}{|[x]_{AT}^{\geq}|} = \frac{|[x]_{AT}^{\geq}|}{|[x]_{AT}^{\geq}|} = 1,$$

i.e., $\mu_{X^c}(x) = 1 - \mu_X(x)$. Thus, for any $x \in U$, one can obtain that $\mu_X(x)(1 - \mu_X(x)) = \mu_{X^c}(x)(1 - \mu_{X^c}(x))$, i.e., $E(F_X^{AT}) = E(F_{X^c}^{AT})$.

Assume that the decision attribute d makes a partition of U into a finite number of classes; let $\mathbf{D} = \{D_1, D_2, \cdots, D_r\}$ be a set of these classes that are ordered, that is, for all $i, j \leq r$ if $i \geq j$, then the objects from D_i are preferred to the objects from D_j. The sets to be approximated are an upward union and a downward union of classes, which are defined as $D_i^{\geq} = \bigcup_{j \geq i} D_j$, $D_i^{\leq} = \bigcup_{j \leq i} D_j$, $(i \leq r)$ [15, 16]. The statement $x \in D_i^{\geq}$ means "x belongs to at least class D_i", whereas $x \in D_i^{\leq}$ means "x belongs to at most class D_i". In the following, we review the definitions of the lower and upper approximations of $D_i^{\geq}$ $(i \leq r)$ with respect to the dominance relation $R_C^{\geq}$ in an ODT [20].

Definition 5.[15, 16] Let $S = (U, C \cup d)$ be an ODT, $A \subseteq C$ and $\mathbf{D} = \{D_1, D_2, \cdots, D_r\}$ the decision induced by d. Lower and upper approximations of $D_i^{\geq}$ $(i \leq r)$ with respect to the dominance relation $R_C^{\geq}$ are defined as

$$\underline{R_C^{\geq}}(D_i^{\geq}) = \{x \in U \mid [x]_C^{\geq} \subseteq D_i^{\geq}\},\ \overline{R_C^{\geq}}(D_i^{\geq}) = \bigcup_{x \in D_i^{\geq}} [x]_C^{\geq}.$$

Denoted by $\underline{R_C^{\geq}}(\mathbf{D}) = (\underline{R_C^{\geq}}(D_1^{\geq}), \underline{R_C^{\geq}}(D_2^{\geq}), \cdots, \underline{R_C^{\geq}}(D_r^{\geq}))$, $\overline{R_C^{\geq}}(\mathbf{D}) = (\overline{R_C^{\geq}}(D_1^{\geq}), \overline{R_C^{\geq}}(D_2^{\geq}), \cdots, \overline{R_C^{\geq}}(D_r^{\geq}))$. $(\underline{R_C^{\geq}}(\mathbf{D}), \overline{R_C^{\geq}}(\mathbf{D})$ are called the rough decision induced by C. For any object $x \in U$, the membership function of x in $\mathbf{D}$ is defined as

$$\mu_{\mathbf{D}}(x) = \frac{|[x]_C^{\geq} \cap D_j^{\geq}|}{|[x]_C^{\geq}|} \quad (u \in D_j), \tag{6}$$

where $\mu_{\mathbf{D}}(x)$ $(0 \leq \mu_{\mathbf{D}}(x) \leq 1)$ represents a fuzzy concept. It can generate a fuzzy set $F_{\mathbf{D}}^C = \{(x, \mu_{\mathbf{D}}(x)) \mid x \in U\}$ on the universe U.

Definition 6. Let $S = (U, C \cup d)$ be an ordered information system and $\mathbf{D} = \{D_1, D_2, \cdots, D_r\}$. A fuzziness measure of a rough decision is defined as

$$E(F_{\mathbf{D}}^C) = \sum_{i=1}^{|U|} \mu_{\mathbf{D}}(x_i)(1 - \mu_{\mathbf{D}}(x_i)), \tag{7}$$

where $\mu_{\mathbf{D}}(x_i)$ denotes the membership function of $x_i \in U$ in the decision $\mathbf{D}$.

Example 2. (Continued from Example 1.) Suppose that $D_1 = \{x_2, x_4, x_6\}$ and $D_2 = \{x_1, x_3, x_5\}$. From formula (6), we have that $\mu_{\mathbf{D}}(x_1) = \frac{|[x_1]_C^{\geq} \cap D_2^{\geq}|}{|[x_1]_C^{\geq}|} = 1$, $\mu_{\mathbf{D}}(x_2) = \frac{|[x_2]_C^{\geq} \cap D_1^{\geq}|}{|[x_2]_C^{\geq}|} = \frac{2}{3}$, $\mu_{\mathbf{D}}(x_3) = \frac{|[x_3]_C^{\geq} \cap D_2^{\geq}|}{|[x_3]_C^{\geq}|} = 1$, $\mu_{\mathbf{D}}(x_4) = \frac{|[x_4]_C^{\geq} \cap D_1^{\geq}|}{|[x_4]_C^{\geq}|} = 1$, $\mu_{\mathbf{D}}(x_5) = \frac{|[x_5]_C^{\geq} \cap D_2^{\geq}|}{|[x_5]_C^{\geq}|} = 1$, $\mu_{\mathbf{D}}(x_6) = \frac{|[x_6]_C^{\geq} \cap D_1^{\geq}|}{|[x_6]_C^{\geq}|} = 1$. Therefore, $E(F_{\mathbf{D}}^C) = \sum_{i=1}^{6} \mu_{\mathbf{D}}(x_i)(1 - \mu_{\mathbf{D}}(x_i)) = 1 \times (1-1) \times 5 + \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$.

Proposition 5. In an ordered decision table $S = (U, C \cup d)$, the fuzziness measure of a crisp decision equals zero.

Proof. Let $\mathbf{D} = \{D_1, D_2, \cdots, D_r\}$ be a crisp decision in the ordered decision table, i.e., $\underline{R_C^{\geq}}(D_j^{\geq}) = \overline{R_C^{\geq}}(D_j^{\geq})$, $j = \{1, 2, \cdots, r\}$. Hence, for any one has that $[x]_C^{\geq} \subseteq D_j^{\geq}$. Thus, $\mu_{\mathbf{D}}(x) = \frac{|[x]_C^{\geq} \cap D_j^{\geq}|}{|[x]_C^{\geq}|} = \frac{|[x]_C^{\geq}|}{|[x]_C^{\geq}|} = 1$, $\forall x \in U$. Therefore, $\mu_{\mathbf{D}}(x_i)(1 - \mu_{\mathbf{D}}(x_i)) = 0$, $i \leq |U|$, i.e., $E(F_{\mathbf{D}}^C) = 0$. This completes the proof.

5 Conclusions

In this study, we have constructed the membership function of an object through using the dominance class including itself. Based on the membership function, we have introduced a consistency measure to calculate the consistency of an ordered decision table and fuzziness measures to compute the fuzziness of an ordered rough set and an ordered rough classification in the context of ordered information systems. Their mechanisms and validity have been shown by several illustrative examples. These results will be helpful for understanding the uncertainty in ordered decision tables.

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